100 Problems each involving the Number 100

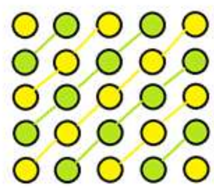
James Tanton

In this whirlwind of a talk we offer 100 problems each involving the number 100 that have been used to inspire mathematical musing among students, educators, and general mathematical enthusiasts and circlers.

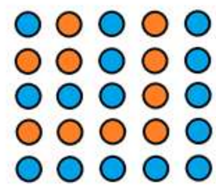
Arithmetic:

1. (Gauss?) What is the sum of the numbers from 1 up to 100?
   What is the sum of the first 100 even numbers?
   What is the sum of the first 100 odd numbers? (What is the 100^{th} odd number?)

   What's $1+2+3+\cdots+99+100+99+\cdots+3+2+1$?
   (And so, again, what is $1+2+3+\cdots+100$?)

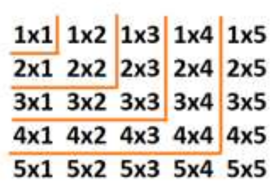


1+2+3+4+5+4+3+2+1
=25



1+3+5+7+9
= 25

2. What is the sum of all the 100 products in a ten-by-ten multiplication table?
3. What is the sum of the first 100 cube numbers?

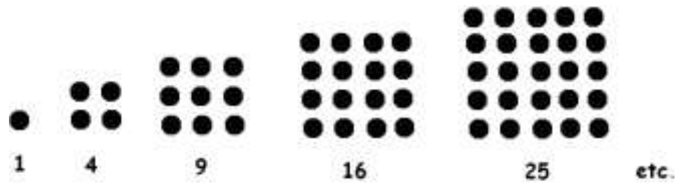


$1^3+2^3+3^3+4^3+5^3$

4. What is the sum of the first 100 square numbers? What is the sum of the first k th powers?
   What is the sum of the first 100 triangle numbers?

5. One hundred students sit in a circle, in chairs, one student per chair. Is it possible for each student to move a some number of seats clockwise, so that one student moves 1 place over, one student moves 2 places over, one student 3 places over, and so on, all the way up to one student moving 100 places over (to return to his original seat)? We want the final arrangement of students to again be one student per chair.

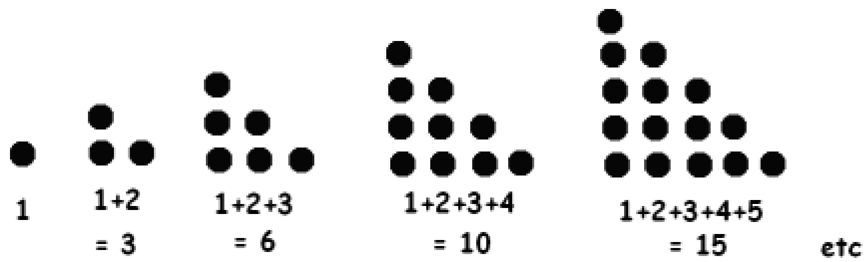
6. The sequence of square numbers begins 1, 4, 9, 16, 25, … and the n-th square number is n^2.



The sequence of non-square numbers begins 2, 3, 5, 6, 7, 8, 10, 11, 12, 13, 14, 15, 17, ….

What is the 100th non-square number?

7. The sequence of triangular numbers begins 1, 3, 6, 10, 15, … and the n-th triangle number is $\frac{1}{2}n(n+1)$.



The sequence of non-triangular numbers begins: 2, 4, 5, 7, 8, 9, 11, …

What is the 100th non-triangular number?

8. **Extension:** If $\langle x \rangle$ denotes the real number x rounded to the nearest integer, then the nth non-square number is $\langle n+\sqrt{n} \rangle$ and the nth non-triangular number is $\langle n+\sqrt{2n} \rangle$. Establish these formulas.
(What non-numbers are given by $\langle n+\sqrt{3n} \rangle$, $\langle n+\sqrt{4n} \rangle$, and so on?)

9. Take fifty numbers from the set $\{1,2,3,\ldots,100\}$ and arrange them in a row in increasing order. Arrange the remaining numbers in decreasing order in a row beneath them. Take the positive difference between the numbers in each column and add all fifty differences. Prove that this sum is sure to equal 50^2. (*Proizvolov's Identity.*)

2	3	7	9	…	93	95	99	100
98	97	96	94	…	6	5	4	1

96 + 94 + 89 + 83 + …+ 87 + 90 + 95 + 99 = 10000 (TYPO! Make that "= 2500")

10. How many polynomials p with non-negative integer coefficients are there satisfying $p(10)=100$?

11. A polynomial q with integer coefficients has $q(1)$, $q(2)$, …, $q(100)$ each a multiple of 100. Prove that $q(n)$ is a multiple of 100 for every integer n.

    Reference: See www.jamestanton.com/wp-content/uploads/2012/03/Cool-Math-Essay_NOVEMBER-2014_Polynomial-Fun1.pdf for a selection of polynomial puzzles.

12. Ninety-nine of the numbers 1 through 100 will be read out loud to you, one at a time, chosen in some random order. You know you will be asked afterwards which number was skipped. What is a good strategy for handling this task asked of you?

Averages:

13. A list of 100 numbers has the property that the average value of the first 99 numbers in the list equals the average of all 100 numbers. What can you say about the 100th number in the list?

14. **Extension:** Let A be the average of the first 37 numbers in a list of 100 numbers, B the average of the remaining 73 numbers, and C the average of all 100 numbers. If two of A, B, and C are equal in value, what can you say about the third value?

15. One hundred students sit in a circle in such a way that each child has age equal to the average of ages of his or her two immediate neighbours. Betty and Luke sit in the circle. Betty is 10 years old. How old is Luke?

16. **Extension:** Suppose, instead, each child has age the average of the age of the person two places to her left and three places to her right. If Betty is 10, how old is Luke?

    Variation: Each student has age the average age of the three people immediately two her left and the person seven places to her right.

    Research: What patterns of "averages" work for this question?

17. Darian is about to write one hundred numbers in the cells of an 10-by-10 grid of squares, one number per cell, in such a way that each number equals the average of its horizontal and vertical neighbors. (Corner cells have two neighbors. Edge cells have three neighbors. Interior cells have four.) For example, if Darian writes 7 in the top left cell, he could then write 4 and 10, say, in the two neighboring cells. The cell containing the number 4 could then be bordered by 7 (which is already filled in), 8 and -3, say.

    When Darian finally completes this exercise, what does he notice?

7	10								
4	8								
-3									

18. Can a set of 100 data values have mean 100, median 100, and mode 1000? How about mean 100, median 1000, and mode 100? How about mean 1000, median 100, and mode 100?

    Repeat this question for 101 data values.

Counting:

19. What is the 100th integer that does not contain the digit 0?
    What is the 100th integer that does contain the digit 0?

20. What is the 100th palindrome?
21. A counting number is "balanced" if each digit that appears in the number does so the same number of times. (For example, 11353350005 is balanced.) What is the 100th balanced number?

22. The number 21200 describes itself: it has 2 zeros, 1 one, 2 twos, 0 threes, and 0 fours. What is the 100th self-descriptive counting number?

23. What is the 100th counting number that has precisely two 1s in its base-two representation?

    Extra: What is the smallest number that can be written as a sum of three powers of two in two different ways?

24. What is the 100th counting number that has more 1s than 0s in its base-two representation?

25. What is the final digit of the 100th Fibonacci number in base eight?

    What is the final digit of the 100th Fibonacci number in base ten?

26. **Challenge:** For any prime $p \geq 5$, show that the final two digits of the p^2th Fibonacci number are the same as the final two digits of p^2 itself. (That is, show that $F\left(p^2\right) \equiv p^2 (\mathrm{mod}\, 100)$.)

27. What are the final two digits of the 100th power of two?

28. How many Pythagorean Triples possess the number 2^{100}?

Weird Factorials:

29. With how many zeros does $100!$ end?

30. What is the rightmost nonzero digit of $100!$?

31. **Extension:** The number $\alpha = 0.12642242...$ has nth digit the rightmost nonzero digit of $n!$. Is α rational?

    (Comment: According to OEIS, G. Dresden has proved α is transcendental; 2006.)

32. Prove that the product of one-hundred consecutive integers is sure to be divisible by $100!$.

33. Is $\frac{100!}{100}$ an integer? Is $\frac{100!}{100^2}$ an integer? Is $\frac{100!}{100^3}$ an integer? How high can we go?

34. **Extension:** For which k are there infinitely many n such that $\frac{n!}{n^k}$ is an integer?

35. What is the largest value of k such that $\frac{100!}{2^k}$ is an integer?

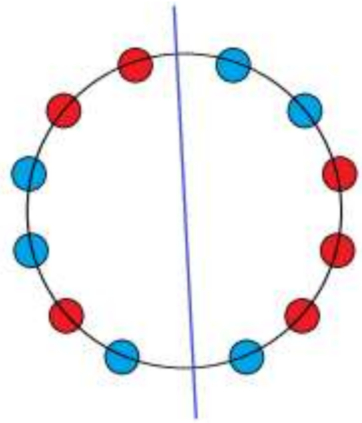
    Aside: Is it true that $\frac{n!}{2^{n-k}}$ is an integer if and only if n is a sum of at most k powers of two?

(For example, $100 = 64 + 32 + 4$ and $\frac{100!}{2^{97}}$ is an integer, but $\frac{100!}{2^{98}}$ is not.)

For which n is $\frac{(2n)!}{3^{n-1}}$ an integer.

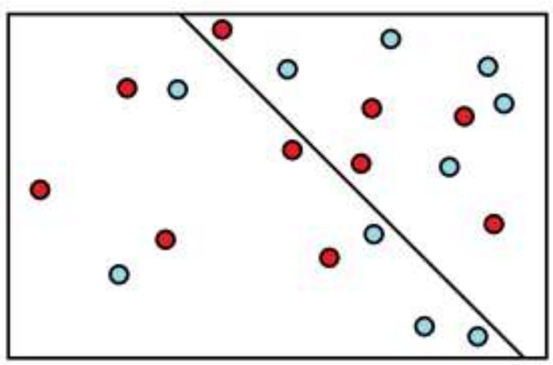
Separating dots:

36. Fifty red beads and fifty blue beads are placed in a circle in a necklace. Is it certain to be possible to cut the necklace in just two places to obtain two pieces each with twenty-five beads of each color?



37. One hundred dots are drawn on a page. Is there sure to be a single straight line one can draw across the page that divides to the group into fifty dots on one side of the line and fifty on the other?

38. One hundred dots are drawn on a page, fifty colored red and fifty colored blue. No three dots are collinear. Is there sure to be a single straight line one can draw across the page so that twenty-five dots of each color lie on either side of the line? (Is the "no three dots are collinear" condition necessary?)



39. One hundred dots on drawn on the surface of a sphere. Is there sure to be a great circle that separates the dots into fifty dots within each of two hemispheres?

40. Is there also a spherical analog to question 38?

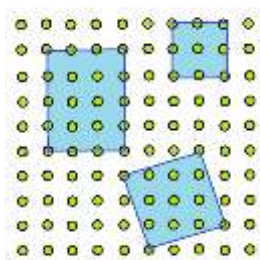
Combinatorics:

41. What is the smallest number of cuts needed to cut a 10 inch by 10 inch square of paper into one-hundred unit squares? Assume you can stack pieces paper on top of another between cuts and cuts that slice through several layers of paper simultaneously count as just one cut.

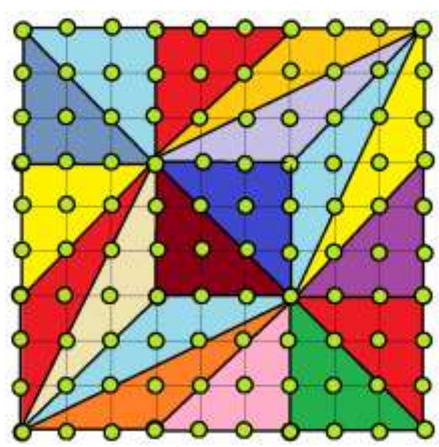
    What is the smallest number of cuts needed to cut a 2 inch by 50 inch rectangle of paper into one-hundred unit squares? A 4 inch by 25 inch rectangle of paper? Again assume you can stack pieces paper on top of one another between cuts.

    How do the answers to the previous questions change if you are also allowed to fold paper between cuts?

42. How many rectangles can one find in a 10-by-10 square array of dots?
    How many (non-tilted) squares can one find in a 10-by-10 square array of dots?
    How many tilted squares can one find in a 10-by-10 square array of dots?



43. Draw a 10-by-10 square array of dots. The four corner squares define a large square of area 81 square units. It is possible to subdivide this square into an even number of triangles, each of the same area, with vertices landing on grid dots.



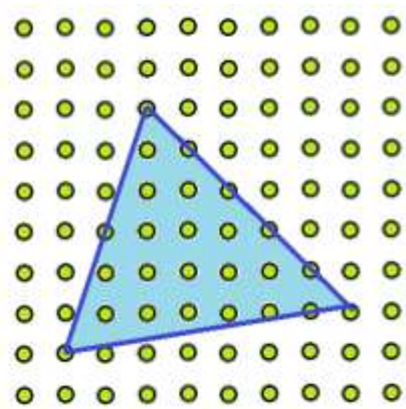
    Is it possible to subdivide the large square into an odd number of triangles of equal area with vertices landing on grid dots?

    Reference: See www.jamestanton.com/wp-content/uploads/2012/03/Cool-Math-Essay_SEPTEMBER-2014_Dividing-Squares-into-Triangles1.pdf.

44. It is impossible to draw an equilateral triangle in the coordinate plane with each vertex a point possessing integer coordinates. (Why?) Thus, in a 10-by-10 array of dots it is impossible to draw an equilateral triangle with each vertex lying at the exact center of a dot. But dots have finite width!

    Suppose each dot in a 10-by-10 is centered about a point with integer coordinates and is a circular disc of radius $r = 0.1$ units. Is it possible to draw an equilateral triangle so that each of its vertices lands inside a distinct dot?

45. In question 39, is there a smallest value of r for which this feat can be accomplished?



46. One-hundred cups sit upside down on a table. By turning two cups over at a time (call this a "move") it is clearly possible to set all the cups upright. The same feat can be accomplished if a move is now defined as turning four distinct cups over simultaneously.

    Can the feat be accomplished if a move is set as turning three distinct cups over simultaneously? (During plays of these games the individual cup could be inverted several times as part of several different moves.)

    Research: If a move is now set as inverting k distinct cups simultaneously, for which k can the feat be accomplished?

47. One hundred great circles are drawn on a sphere in such a way that only two circles ever intersect at an intersection point. Into how many regions do these circles divide the sphere? How many intersection points are there in total?

48. One hundred distinct straight lines are drawn across a page. What is the maximum number of regions they could divide the page? What is the minimum number? For every integer between the maximum and minimum values, is there a pattern of 100 lines across the page dividing the page into that many regions?

49. One hundred dots are drawn on the boundary of a disc and chords are drawn connecting each and every pair of dots. The dots are placed about the rim in an unsymmetrical fashion to ensure that only two chords ever meet at an intersection point.

Explain why the total number of chords drawn must be ${}_{100}C_2$.

Explain why the total number of intersection points made must be ${}_{100}C_4$.

Show that the disc is divided into $1+{}_{100}C_2+{}_{100}C_4$ regions.

Comment: One shouldn't give the answers away in the question statement. See the video www.jamestanton.com/?p=775 for more on this puzzle.

50. One hundred coins are placed in a row. Some coins are heads up and some coins a tail up. In a game of solitaire a "move" is conducted by removing a heads up coin and flipping the coin immediately to its left (if there is one) and the coin immediately to its right (if there is one). One keeps the spaces of missing coins in place. This means that throughout play of this game immediate neighbors of coins could be absent.

The goal of the solitaire game is to remove all one hundred coins.

Develop a winning strategy for the game if, initially, there are 37 heads up coins in the row. Explain why one shouldn't bother playing the game if, initially, there are 50 heads up coins in the row.

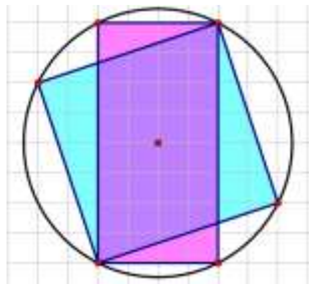
In general, which counts of heads up coins in the row initially can lead to a certain win, and which must lead to a certain loss?

51. One hundred red dots are placed on a line and one hundred blue dots are placed on a parallel line. Line segments are then drawn between the parallel lines each connecting a blue dot to a red dot. Each and every blue dot is connected to each and every red dot with a line segment. Assuming that the dots have been placed in a non-symmetrical manner so that only two line segments meet at a point of intersection, how many intersection points are there in total? (Also, how many line segments were drawn and into how many regions is the space between the parallel lines divided?)

Geometry:

52. Consider a 1×100 rectangle and a square inscribed in a circle. What is the ratio of the areas of these two quadrangles? (Find a nice geometric way to see the answer.)

    Repeat the previous question for all rectangles with integer side-lengths of area 100 inscribed in a circle.

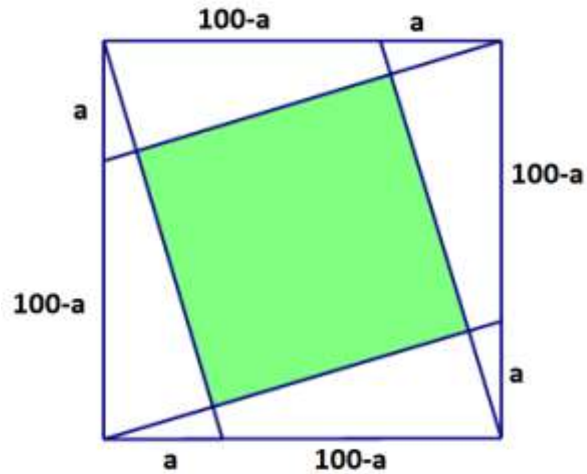


UNRELATED ASIDE: FOUND MATH!

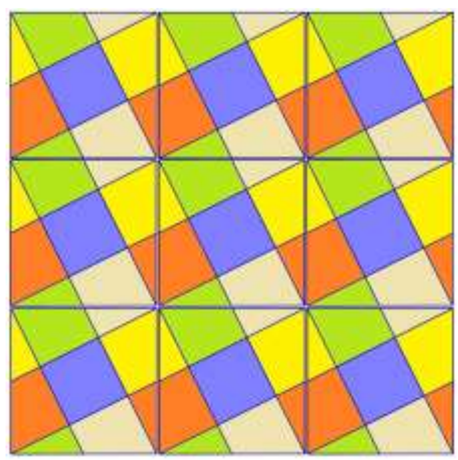
Phoenix Airport

Two intersecting chords in a circle divide each other into lengths with the same product. (Prove this!)

53. In a 100-by-100 grid of squares a line is drawn from each vertex to a point a distance $\frac{a}{100}$ along an opposite side as shown. What is the area of the interior square they form?



Comment: Start with lines connecting the midpoint of each opposite side ($a = 50$). Make a minimum of nine cardboard copies of the squares with these lines drawn in and tile the floor with squares. We then see that the area of the middle square is one-fifth the area of the entire square.

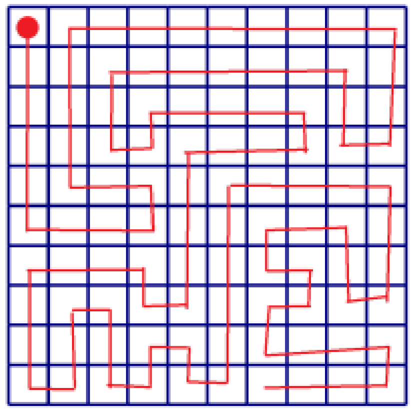


Can one use tiling arguments to answer the original question for other values of a ?

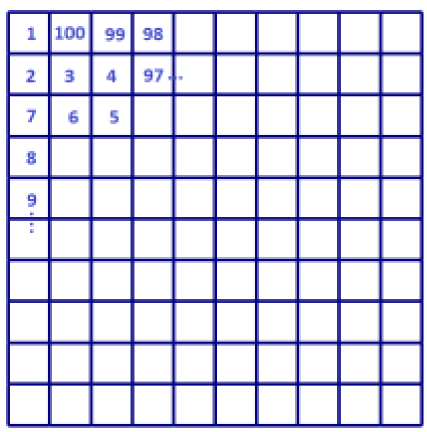
Extension: Explore analogous results for lines drawn in triangles. (Tiling can be of help here too.)

Topology?

54. In a 10-by-10 grid of squares, starting at the top left cell, it is possible to move from cell to neighboring cell via vertical and horizontal steps to visit each cell of the grid exactly once. Can such a feat be accomplished starting from other cells of the grid as well?



55. In a 10-by-10 grid of squares I start at the top left square and move from cell to neighboring cell via vertical and horizontal steps to return to the top left cell. If I number the cells 1, 2, 3, 4, … as I go, which cells could have number 17 ?



56. In a 10-by-10 grid of squares I start at the top left square and move from cell to neighboring cell via vertical and horizontal steps to return to the top left cell. If I number the cells 1, 2, 3, 4, … as I go, could two adjacent cells differ by 50 in their assigned values?

57. A lattice point in the plane is a point with integer coordinates. If I walk a 100 unit steps from lattice point to lattice point and return to start, must there by some four cells I visited that form the vertices of a (possibly titled) square?

58. Which of the numbers $\frac{1}{100}, \frac{2}{100}, \ldots, \frac{99}{100}$ are in the Cantor set?

59. A 10 foot by 10 foot square of paper is folded in some haphazard way into a 1 foot by 1 foot square, one hundred layers thick. A single straight cut through all one hundred layers is made across the one-foot unit square. How many pieces of paper could result?

60. Design a method of linking together 100 loops of string so that if any one string is cut, the remaining 99 loops separate into 99 unlinked rings.

Associativity and Commutativity

61. The numbers 1 through 100 are written in a board. In a game of solitaire, a move consists of erasing two numbers on the board and replacing them their sum plus their product:

$$a, b \to a + b + ab .$$

After 99 moves one number will be left on the board.

Explore.

Comment: Astoundingly, the final number in any play of this game is the same. First playing this game with a small selection of initial numbers can lead participants to this as a conjecture.

62. **Variation:** To facilitate thinking about puzzle 61 consider the variation given by:

$$a, b \to a + b + 1 .$$

Is it clear in this game that the final answer is unique? (What is that answer?)

63. If you are not afraid of fractions, play the game of question 61, replacing any two numbers a and b on the board with $\frac{ab}{a+b}$. After 99 moves, what is the final unique number on the board?

64. If you like Pythagoras, play the game of question 61, replacing any two numbers a and b on the board with the length of hypotenuse of a right triangle with legs a and b. After 99 moves, what is the final unique number on the board?

65. If you are like logarithms, play the game of question 61, replacing any two numbers a and b on the board with $\log\left(10^a + 10^b\right)$. After 99 moves, what is the final unique number on the board?

66. If you like big numbers, play the game of question 61, replacing any two numbers a and b on the board with $(a+88)(b+88)-88$. After 99 moves, what is the final unique number on the board?

67. If you like common divisors, play the game of question 61, replacing any two numbers a and b on the board with $\mathrm{gcd}(\mathrm{a}+1, \mathrm{b}+1)-1$. After 99 moves, what is the final unique number on the board?

Or try any of these crazy variations:

68. $a,b \to \frac{ab}{\sqrt{a^2+b^2}}$

69. $a,b \to \left(1+\sqrt{a}+\sqrt{b}\right)^2$

70. $a,b \to \sqrt{a^2+a^2b^2+b^2}$

71. $a,b \to \sqrt[3]{a^3+a^3b^3+b^3}$

72. What is really going on with questions 61-71? (See www.jamestanton.com/wp-content/uploads/2012/03/Cool-Math-Newsletter_May2012.pdf .)

Parity

73. Quentin ripped out 100 pages, each chosen at random, from Poindexter's textbook. He never looked at the pages he removed but stated to Poindexter with absolute certainty "The sum of the missing page numbers from your book is even." Quentin is right. How did he know?

74. Can you make change for a dollar using a combination of fifteen pennies, dimes, and quarters?

75. The numbers 1 through 100 are written in a row on the board. In a game, Agatha and Beatrice take turns placing either a $+$ sign or a $-$ sign between a pair of neighboring numbers. After 99 turns, they compute arithmetic sum on the board. If the final answer is an odd number, Agatha wins. If it is an even number, Beatrice wins. Agatha goes first. What first move should she make in order to win?

76. A 10 foot by 10 foot square of paper, black on one side and white on the other, is folded in some haphazard way into a 1 foot by 1 foot square, one hundred layers thick. The folded stack of layers is placed on the ground. How many of those layers have a black face facing upwards?

77. One hundred circles and one hundred squares are drawn on a board. In a game, Mildred and Mable take turns erasing any two symbols from the board and replacing them with a single symbol given by the following rule:

    If two identical shapes are erased, draw a square; otherwise draw a circle.

    After 199 moves a single symbol remains on the board. If that symbol is a circle, Mildred wins. If it is a square, Mable wins.

    What is the best strategy of play for Mable? (Should she go first or second?)

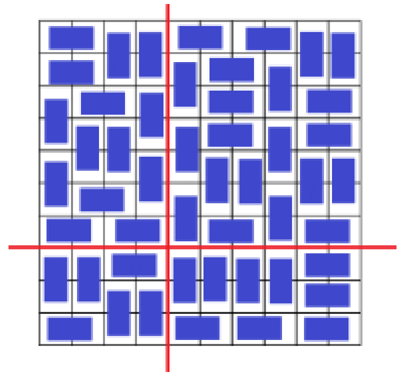
78. In a game of solitaire, Allistaire writes the numbers 1 through 100 on a board. He then erases two numbers and replaces them by their positive difference. After 99 such moves, he'll be left with a single number on the board. If that single number is lucky 7, he wins his game of solitaire. How should he play to win?

79. In a certain island county there are 100 roads. Each road connects two towns in the county and, according to county records, each town has seven roads emanating from it. Can county records possibly be correct?

80. A *unit fraction* is a fraction of the form $\frac{1}{a}$ with a a positive integer. We say $\frac{1}{a}$ is an *odd unit fraction* if a is an odd integer. Can the number 1 be written as a sum of one hundred odd unit fractions?

81. Can the fraction $\frac{1}{100}$ be written as a sum of odd unit fractions?

82. The following tiling of a 10-by-10 grid of squares with 50 dominos contains both a vertical and a horizontal “fault line.”



Find a tiling of a 10-by-10 grid with no fault lines at all.

Find a tiling of a 4-by-25 grid of squares with no fault lines.

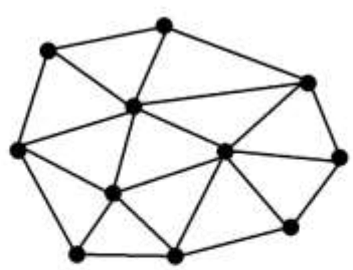
83. Following question 82: What is the maximum number of fault lines a tiling of a 10-by-10 grid of squares could possess? What is the least number a tiling of a 4-by-25 grid of squares must possess?

84. One hundred great circles are drawn on the surface of a sphere. Prove that the resulting map on the globe can be colored using just two colors in such a way that any two neighboring regions sharing a common section of arc of positive length are assigned different colors.

85. Draw one hundred curly intersecting and self-intersecting lines cross a page. Under which conditions is the resulting map sure to be two-colorable?

86. Draw some dots inside a 100-sided polygon. Connect pairs of dots with line segments, interior dots with vertex points too, to make triangles that completely subdivide the polygon into triangles. (Make sure each triangle in the subdivision really has only three dots on its boundary.)

Is it possible to subdivide the 100-gon into 100 sub-triangles via this method? Less than 100 sub-triangles? Into 120 sub-triangles? Into 1002 sub-triangles? Into 2001 sub-triangles?



Reference: These puzzles are classic. Some appear in Fomin, Genkin, and Itenberg’s gem *Mathematical Circles (Russian Experience)* (AMS 1996).

Pile Splitting:

87. Tycho has 100 pebbles in a pile. He splits the group into two piles: one of 20 pebbles and one of 80 pebbles, say, and writes on a piece of paper the product $20 \times 80 = 1600$. He then takes the pile of 20 pebbles and splits it into a group of 3 and 17 and writes the product $3 \times 17 = 51$ on the paper. He splits the other pile into a group of 70 and 10, adding the product $70 \times 10 = 700$ to his list. Tyhco repeats this process, taking a pile and splitting it into two groups, one of a pebbles and another of b pebbles, say, and writing the product $a \times b$ on his paper. He does this until he has 100 piles each containing just one pebble.

    Tycho then adds all the products we wrote down on his paper: $1600 + 51 + 700 + \cdots$. Explain why he is sure to get the sum 4950.

88. How many products did Tycho write down on his paper?

89. Explain why exactly fifty of the products in Tycho's sum are odd.

90. For each pile split into a group of a pebbles and a group of b pebbles, instead of writing the product $a \times b$ on his paper, Tycho wrote the value $ab(a+b)$, the product of the sum and the product of the two counts. Explain why the sum of all these values is sure to be $\frac{99 \times 100 \times 101}{3}$.

91. For each pile split into a group of a pebbles and a group of b pebbles, instead of writing the product $a \times b$ on his paper, Tycho wrote the product of the a-digit number consisting of nothing but 8s and the b-digit number consisting solely of 8s. (So, in splitting a pile of 5 into 2 and 3, Tycho writes the product 88×888.) Explain why the sum of these products must be 790123456790123456790123456

Classic Gnomes, Prisoners, and Pirates:

94. One hundred gnomes have agreed to play a gruesome game with an evil villain. The gnomes will stand in a line so that each gnome sees the backs of all the gnomes before him. (Thus, the front gnome, call him gnome number 1, sees no other gnomes. The final gnome, gnome number 100, sees the backs of all other 99 gnomes.)

    The evil villain will then place a hat on each gnome's head. The gnomes know that each hat will be either black or white, but they do not know how many hats of each color there shall be in the line. Each gnome will be able see the hats of all the gnomes before him but, alas, not see the color of his own hat (nor the hats behind him).

    Starting with gnome number 100, the gnome at the back of the line, and working along the line to gnome number 1, the evil villain will then ask the gnomes in turn: "What is the color of your hat?" Each gnome is allowed to utter only one word: "black" or "white." If the gnome answers correctly, he lives. If he answers incorrectly, he dies.

    Each gnome will be able to hear the answers spoken behind him, and the subsequent sighs of relief or the cries of horror. No other sounds of communication will be allowed.

    Knowing that they are about to play this game, fully cognizant of all the details just outlined, what scheme could the gnomes agree upon before playing this game to ensure the survival of a maximal number of gnomes? What is that largest survival number?

95. What if gnomes are given either black, white, or rouge hats and are allowed two guesses? One guess?

96. One hundred people line up to take their seats in a 100-seat theatre, but the first person has lost his ticket and chooses a seat randomly. Each person, thereafter, either sits in her correct assigned seat or, if occupied, selects an unoccupied seat at random. What is the probability that the 100^{th} person sits in her correct seat?

97. A jail warden announces to 100 prisoners that she is about to conduct the following experiment:

    Each day she will roll a 100-sided die to select a prisoner at random and then bring that prisoner to a particular room. The room contains a lamp that can be either on or off, and each prisoner, if she wishes, can change the state of the lamp (turn it on if it is off, turn it off if it is on) on her visit. There will be absolutely no communication between the prisoners except via the state of the lamp: on or off.

    She run this experiment for as many days needed until some prisoner announces upon his visit to the room: "I know that each and every one of my prison-mates has made at least one visit to this room."

    If this claim is made and it is correct, all 100 prisoners will be set free. If the claim is incorrect, then all 100 prisoners will be executed.

    Knowing that they are about to take part in this experiment and given the opportunity to confer before being separated, what scheme could the prisoners devise to ensure their freedom?

98. Ten pirates, ranking from number 1, the Captain, to number 10, the cabin boy, come across a treasure of 100 gold coins. They decide to divvy the coins according a voting scheme. First, the cabin boy will suggest how the share out the coins and the pirates will then vote whether or not to adopt that scheme. If more than 50% of the pirates agree to the scheme, they will follow it. If not, the cabin boy will be thrown overboard and pirate number 9 will then suggest a scheme for sharing the coins among the remaining nine pirates. If in a vote more than 50% agree, they will adopt it. Otherwise pirate number 9 is thrown overboard and the suggesting/voting scheme is repeated, with pirate number 8, and so on, possibly down to pirate number 1.

    Assuming that each pirate is a greedy, but logical, thinker and will not vote for the demise of another pirate if he realizes that he will end up with less coins as a result, what sharing scheme could the cabin boy suggest to ensure his survival?

99. One hundred people on an island, numbered 1 through 100, are playing the game of "survivor." They will vote on whether or not player 100 should stay on the island or leave. A two-thirds or more vote will ensure that player 100 remains and hence that all 100 people stay on the island. If player 100 does not receive at least two-thirds of the vote, she will be taken off the island and the remaining 99 players vote will then vote on whether or not play 99 should stay, looking for at least a two-thirds majority.

    They continue this way until some player succeeds in being voted to stay. (And at this point the voting stops and all surviving players stay on the island.)

    Will player 100 stay on the island? (Assume that each player is a rational thinker, wants to stay on the island, and will vote for herself to stay.)

100. **Variation:** How does the answer change if one must obtain strictly more than two-thirds of the votes in order to stay?

Research: Explore the results of these two questions for other voting percentage.

References: There are many puzzles about "100 prisoners" or "100 pirates" bandied about on the internet. I explain the gnome puzzle (question 94) in www.jamestanton.com/?p=914 . Peter Wrinkler has variations of question 96 and 97 (and many other really good juicy puzzles) in his book *Mathematical Puzzles: A Connoisseur's Collection* (A K Peters, 2004). I heard about the pirates and their coins puzzle (question 98) when it made the email rounds across mathematics departments in 2001. The "survivor" variation is a simplification of it, leading to some curious research questions.